

A HAMILTONIAN MODEL FOR MULTIPLE PRODUCTION IN HADRON-HADRON COLLISIONS

G. CALUCCI, D. TRELEANI

*Dipartimento di Fisica teorica, Università di Trieste, Strada Costiera 11. I-34014
and I.N.F.N. Sezione di Trieste, Italy*

A Hamiltonian eikonal model for the multiple production in high energy hadron hadron collisions is presented and worked out in the aim of providing a simple frame for many different observables of these processes. The Hamiltonian formulation ensures that it has the unitarity built in by construction and the eikonal approximation makes easier the discussion of the possible spacial inhomogeneities of the hadrons. The experimental data which are examined are the inelastic cross section and the single and double inclusive cross sections.

1 Description of the model

A Hamiltonian model for the description of the multiple production process in hadron-hadron collision is presented with the main aim of bringing together different observables within a unique frame. A particular attention is given to those features of the inelastic processes that can give informations on the proton structure.

The physical ingredients of this model are the following:

A sharp distinction is made between the soft dynamics, which provides the binding of the partons in the hadrons and also the final hadronization of the shaken-off partons and the hard dynamics that causes the parton scattering. The hard collision gives a finite transverse momentum to the partons, which remains however small with respect to the typical longitudinal momentum. Hard rescattering is included, but not hard branching of the partons. The discrete quantum numbers spin and colour are not taken into account.

The model describes the hadrons as sets of bound partons which, due to the interaction, may become finally unbound and show up as jets; the hadronization process is not described.

The only detailed kinematics is the transverse one, the whole treatment lies within the frame of the eikonal formalism.[1,2,3] The sharp distinction between backward and forward degrees of freedom allows the following formulation: There are operators for the bound backward and forward partons a_b, a_f and operators for the unbound partons c_b, c_f . Both are local in \mathbf{b} , the transverse impact parameter of the parton, and they have the standard commutation relations, where every backward operator commutes with every forward operator and every a commutes with every c ; so we can write a free

Hamiltonian:

$$\mathcal{H}_o = \sum_{v=f,b} \omega \int d^2b [a_v^\dagger(\mathbf{b})a_v(\mathbf{b}) + c_v^\dagger(\mathbf{b})c_v(\mathbf{b})] \quad (1)$$

The interaction that we want to describe is the hard collision of two bound partons that give rise to two unbound partons in such a way however that they keep their property of being either backward or forward. Thus the interaction Hamiltonian is written as:

$$\mathcal{H}_I = \lambda \int d^2b h_b(\mathbf{b})h_f(\mathbf{b}) \quad h_v(\mathbf{b}) = c_v^\dagger(\mathbf{b})a_v(\mathbf{b}) + a_v^\dagger(\mathbf{b})c_v(\mathbf{b}) . \quad (2)$$

With this choice it results $[\mathcal{H}_o, \mathcal{H}_I] = 0$, but the theory is not trivial even though it has been much simplified, the S-matrix can be written in the form $\mathcal{S} = \exp[-i\mathcal{H}\tau]$ where τ is an interaction time.

An alternative, and more realistic, form gives a finite size to the hard interaction and a discretization of the transverse plane. The size Δ is related to the cut-off in the transverse momentum that must be put in order to be allowed to perform perturbative calculations, so the natural choice is $\Delta \approx 1/p_\perp^2$; this choice lead also to the interpretation of $\tau \approx 1/p_\perp \approx \sqrt{\Delta}$. The commutation relations become $[A_{v,j}, A_{u,i}^\dagger] = \delta_{i,j}\delta_{u,v}$, and so on. In this way we get:

$$\mathcal{H}_o = \sum_{v,j} \Omega [A_{v,j}^\dagger A_{v,j} + C_{v,j}^\dagger C_{v,j}] \quad (3)$$

$$\mathcal{H}_I = (g/\sqrt{\Delta}) \sum_j H_{b,j} \cdot H_{f,j} \quad H_{v,j} = C_{v,j}^\dagger A_{v,j} + A_{v,j}^\dagger C_{v,j} . \quad (4)$$

The coupling constant g is dimensionless, it is related to the previous coupling constant by $g = \lambda/\sqrt{\Delta}$. Correspondingly the S-matrix is

$$\mathcal{S} = \prod_j \mathcal{S}_j \quad , \quad \mathcal{S}_j = \exp[-i(g/\sqrt{\Delta})\tau H_{b,j} \cdot H_{f,j}] , \quad (5)$$

with the previous interpretation of τ , we get the simpler form:

$$\mathcal{S}_j = \exp[-ig H_{b,j} \cdot H_{f,j}] . \quad (6)$$

In order to apply this model one must choose a definite initial state; it will be factorized in the same way as the S-matrix: as far as its structure in a site j is concerned there are no strong indications. A possible choice, related

to some theoretical ideas about the nonperturbative partonic structure of the hadrons, [4,5] is a local coherent state, so we write

$$|I\rangle = \prod_j |I\rangle_j \quad , \quad |I\rangle_j = \exp[-(|F_b|^2 + |F_f|^2)/2] \exp[F_b A_b^\dagger + F_f A_f^\dagger] | \rangle_j \quad (7)$$

It has to be noted that the weight F of the coherent state may vary from site to site. For simplicity the index j will be not written out, whenever possible.

It is easier to express $|I\rangle_j$ and especially to perform the subsequent calculations in the basis generated by the auxiliary operators P and Q .

$$P = (C + A)/\sqrt{2} \quad Q = (C - A)/\sqrt{2} . \quad (8)$$

In terms of them one gets

$$\mathcal{H}_o = \sum_{v,j} \Omega [P_{v,j}^\dagger P_{v,j} + Q_{v,j}^\dagger Q_{v,j}] \quad , \quad H_{v,j} = P_{v,j}^\dagger P_{v,j} - Q_{v,j}^\dagger Q_{v,j} \quad (9)$$

2 Some results of the model

2.1 Inelastic cross section

In the discrete formulation for the states and for the S-matrix the inelastic cross section is now calculated. In the basis generated by the operators P and Q the operator \mathcal{S}_j is diagonal, so it is easy to calculate the matrix element $S_j = \langle j < I | \mathcal{S}_j | I \rangle_j$, it has the expression:

$$S_j = N^2 \sum_{k_1 \dots k_4} \frac{1}{k_1! k_2! k_3! k_4!} (|F_b|^2/2)^{k_1+k_2} (|F_f|^2/2)^{k_3+k_4} \exp[-ig(k_1-k_2)(k_3-k_4)] . \quad (10)$$

The indices k_1, k_2, k_3, k_4 refer to the quanta created by $P_b^\dagger, Q_b^\dagger, P_f^\dagger, Q_f^\dagger$ respectively. The normalizing factor is $N = \exp[-(|F_b|^2 + |F_f|^2)/2]$. Using the representation:

$$\exp[-ig(k_1-k_2)(k_3-k_4)] = (2\pi)^{-1} \int dudv \exp[iuv + i\alpha u(k_1-k_2) + i\beta v(k_3-k_4)] , \quad (11)$$

with $\alpha\beta = g$, the multiple sum in the expression of S_j can be transformed into an integral. In the final result we use the definitions $T_v = |F_v|^2$ and we have:

$$S_j = (2\pi)^{-1} \int dudv \exp[iuv] \exp[-T_b(1 - \cos \alpha u) - T_f(1 - \cos \beta v)] . \quad (12)$$

When the distribution functions F_v do not vary strongly from site to site one can devise a continuum limit. We consider a relation $F \approx f\sqrt{\Delta}$; therefore, if f is not singular, in the expression of S_j the terms $|F|^2$ become small, the exponential in the integral representation can be expanded and integrated term by term with the result:

$$S_j \approx 1 - |F_b|^2 |F_f|^2 (1 - \cos g) + (1/2) |F_b|^2 |F_f|^2 (|F_b|^2 + |F_f|^2) (1 - \cos g)^2 + \dots \quad (13)$$

With the normalization we are using the inelastic cross section at fixed hadronic impact parameter \mathbf{B} is given by

$$\sigma(\mathbf{B}) = 2 < I | (1 - \Re \mathcal{S}) | I > - | < I | (1 - \mathcal{S}) | I > |^2. \quad (14)$$

The product of the matrix elements is expressed as the exponential of the sum of the logarithms and the sum $\Delta \sum_j$ is finally converted into the integration $\int d^2b$ with the final result:

$$\sigma(\mathbf{B}) = 1 - \exp \left[-\hat{\sigma} \int d^2b |f_b(\mathbf{b})|^2 |f_f(\mathbf{b} - \mathbf{B})|^2 + \hat{\sigma}^2 \dots \right] \quad (15)$$

the parameter $\hat{\sigma} = 2\Delta(1 - \cos g)$ has the role of elementary partonic cross section.

The form of the inelastic cross section is quite usual, it contains however the explicit indication of the possible corrections due to rescattering, they will be discussed below, where a nonuniform model of the hadron will be explored. The cross section arises from the integration over the impact parameter, the result depends very much on the properties of the distribution functions f , which appear through their squared absolute values $t_v(\mathbf{b}) = |f_v(\mathbf{b})|^2$, giving the transverse density of bound partons. In a first discussion the distribution is taken to be completely uniform in b by setting:

$$t_b(\mathbf{b}) = \rho_b \vartheta(R - |b|) \quad , \quad t_f(\mathbf{b}) = \rho_f \vartheta(R - |b|) ; \quad (16)$$

elementary geometrical considerations give $|B| = 2R \cos \gamma/2$ with $0 \leq \gamma \leq \pi$. The exponent in the integrand is given by the partial superposition of the two disks. Since the superposition area is $W = R^2(\gamma - \sin \gamma) = \pi R^2 \xi$ The expression for the inelastic cross section is:

$$\sigma_{in} = 2\pi R^2 \int_0^\pi d\gamma \sin \gamma \left[1 - \exp[-\nu \xi] \right], \quad (17)$$

with $\nu = \hat{\sigma} \rho_b \rho_f \pi R^2$.

It is possible to give a simple analytical form for σ_{in} in the two limiting situations of very small or very large ν . In the first case one gets

$$\sigma_{in} = \pi R^2 \cdot \nu . \quad (18)$$

In the second case we can start from the expression

$$\sigma_{in} = 2\pi R^2 [2 - D(\nu)] \quad (19)$$

where the real function $D(\nu)$ defined by eq(17) is monotonically decreasing, for large ν it results $D(\nu) \approx 2(6\nu^2/\pi^2)^{-1/3} \cdot \Gamma(2/3)$ so that the geometrical limit of black disks $4\pi R^2$ is approached.

2.2 Pair and double-pair production

The production of a pair can seen either as production of a backward parton or of a forward parton, the Hamiltonian being fully symmetric, so we can choose, arbitrarily to look at the forward particles; successively we shall investigate how much the rescattering processes may destroy the sharp correlation between backward and forward scattered partons. We start from the computation of the inclusive production from a single site. Straightforward, although lengthy calculations, easier in the basis generated by the operators P and Q , yield the result

$$\langle X_j \rangle = {}_j \langle I | \mathcal{S}_j^\dagger C_f^\dagger C_f \mathcal{S}_j | I \rangle_j = (T_f/2) [1 - \exp[-T_b(1 - \cos(2g))]] . \quad (20)$$

We can now go to the continuum limit, always under the hypothesis of smooth distributions $t_v(\mathbf{b})$; we find in eq(20) the function $\cos(2g)$ instead of $\cos g$, so we must define a quantity $\kappa = (\Delta/2)[(1 - \cos(2g))]$, which is related to $\hat{\sigma}$ in this way: $\kappa = \hat{\sigma} \cdot [1 + \cos(g)]/2$. The two constants coincides evidently for small g , when both are: $\hat{\sigma} = \kappa = g^2 \Delta$, but the rescattering corrections, higher powers in g^2 , are different.

In the smooth continuous limit we expand the exponential of eq. (4.3) and we get the usual expression:

$$X(\mathbf{B}) = \kappa \int d^2b t_f(\mathbf{b}) t_b(\mathbf{b} - \mathbf{B}) \quad (21)$$

and the inclusive one-particle cross section is

$$D_1 = \int X(\mathbf{B}) d^2B \quad (22)$$

Analogously the two-particle inclusive cross section is found to be

$$D_2 = \kappa^2 \int d^2B d^2b d^2b' t_f(\mathbf{b}) t_b(\mathbf{b} - \mathbf{B}) t_f(\mathbf{b}') t_b(\mathbf{b}' - \mathbf{B}); . \quad (23)$$

A ratio of the quantities that have been now calculated which is of phenomenological interest is $\sigma_{\text{eff}} = [D_1]^2/D_2$, [6,7] In the limit of rigid disk, using the geometrical considerations one gets

$$\sigma_{\text{eff}} = \frac{\pi R^2}{1 - 16/(3\pi^2)} \approx 2.2\pi R^2 , \quad (24)$$

The previously calculated expression of σ_{in} is really the hard part of the total inelastic cross section, where as "hard" part we intend the contribution of all the events where at least one hard scattering happens. If we believe that, in going on with the total energy these events become more and more important we would like to have this term not too small with respect to the experimental σ_{in} , which in turn appears to be sizably larger, of about a factor 2, with respect to σ_{eff} , so in this model we would like to approach, even though not reach, the black-hadron, limit which produces $\sigma_{in} = 4\pi R^2$.

2.3 Multiplicity distribution and forward-backward correlations

The distribution of the multiplicities of the produced pairs is calculated by defining the projection operator over the number of free partons. Since we have always the sharp distinction between backward and forward particles we can choose one of the two particle to define the produced pair: to be definite we take the forward particle as signal of the pair production. For a fixed site j the number projector may be expressed as:

$$\mathcal{P}_n = \frac{1}{n!} : [C^\dagger C]^n e^{-C^\dagger C} : \quad (25)$$

The colon indicates the normal ordering of the C -operators. From this definition one can perform calculations whose qualitative results can be stated in this form: when the total production is not very copious the result may be written as the sum of two Poissonian distributions, one reflects the incoming coherent state the other the rescattering effect , if however the production in a single site is high so that the rescattering is very important the resulting expression may be put into the form of a Poissonian distribution times another factor, but this further factor is not a small correction, it changes in essential way the shape of the distribution.

The same qualitative result is obtained by studying the forward-backward correlations; we can calculate both the variance $\Sigma_f = \langle X_f^2 \rangle - \langle X_f \rangle^2$ with $\langle X_f^2 \rangle = \langle I | \mathcal{S}^\dagger C_f^\dagger C_f C_f^\dagger C_f \mathcal{S} | I \rangle$ and the covariance

$\Sigma_{f,b} = \langle V \rangle - \langle X_f \rangle \langle X_b \rangle$ with: $\langle V \rangle = \langle I | \mathcal{S}^\dagger C_f^\dagger C_f C_b^\dagger C_b \mathcal{S} | I \rangle$

The actual calculations show, as expected, that the correlation

$$\kappa_{f,b} = \frac{\Sigma_{f,b}}{[\Sigma_f \Sigma_b]^{1/2}} \quad (26)$$

for small values of T_v , and so for small production, goes to 1, but when T_v becomes very large it goes to zero.

2.4 Non uniform hadrons

We wish now to explore the possibility that the hadron, and its projection over the transverse plane, shows strong inhomogeneities in the matter density. This is represented by assuming that there are black spots, that cover a limited amount of the transverse area, while a much fainter "gray" background fills uniformly the rest of the hadron.

$$|I_b \rangle_j = \left[x \exp[-|F_b|^2/2] \exp[F_b A_b^\dagger] + y \exp[-|G_b|^2/2] \exp[G_b A_b^\dagger] \right] | \rangle \quad (27)$$

Since the two coherent states are not orthogonal the normalization condition is complicated, however if the two thickness are very different the cross term in normalization condition is very small and we are left with $|x|^2 + |y|^2 \approx 1$. The expression for the S-matrix may be given in the form

$$\mathbf{S} = \mathbf{S}_o \mathbf{S}_c \quad , \quad \mathbf{S}_o = S_o^w \quad , \quad w = W/\Delta \quad (28)$$

the factor S_o^w gives the contribution of the background scattering.

When one uses the assumption that $y \ll x$ *i.e.* that the spots cover a small part of the transverse area it is possible to give a simple expression to \mathbf{S}_c

$$\mathbf{S}_c = 1 - wy^2 \Delta [\rho_b(1 - \hat{\sigma} \rho_f) + \rho_f(1 - \hat{\sigma} \rho_b)] \quad (29)$$

It may be noticed that the factor, $wy^2 \Delta$, represents the part covered by black spots within the interacting area of the hadrons at given \mathbf{B} .

The basic elements for calculating the pair and double pair production have already been given. The local production amplitude is the sum of 4 terms, it will be indicated as: $\langle X_j \rangle = [x^4 X_{o,o} + x^2 y^2 (X_{s,o} + X_{o,s}) + y^4 X_{s,s}]$. The first term represents the pure background interaction, the second and third terms

represent the spot background interaction, the fourth term gives the spot-spot interaction. When one consider explicitly the superposition of the two disks it appears that the four terms have the same geometrical properties and we get for σ_{eff} the same expression as in the case of a uniform hadron. So, at first sight it seems that nothing is gained by introducing an inhomogeneity into the hadron, but in fact some new features are present. The expression of σ_{eff} is purely geometrical, it does not contain the parameters x, y . The expression of σ_{in} can be obtained from the S-matrix (see eq.28, 29), so it depends on y , more in general both on x and on y .

In order to give a clear, although unrealistic evidence of this fact we can consider the limit in which the background is so thin that it contribute negligibly in the inelastic cross section, then the S-matrix element, depends only on the spot-spot interaction. In this situation the inelastic cross section goes to zero with y^4 , the corresponding picture would be: few, wholly black spots distributed in a wide and very thin background. The conclusion of this analysis is therefore that in order to have $\sigma_{\text{eff}} < \sigma_{\text{in}}$ the hadron should be compact, *i.e.* without holes or transparent regions and also with quite sharp edges.

3 Further considerations and conclusions

The model allows further derivations that are here only mentioned: it is possible to introduce correlations in \mathbf{b} in the case of nonuniform hadron; it is also possible to treat to some extent the longitudinal degrees of freedom provided the distinction between forward and backward particles is kept valid. Moreover, in some simple cases analytically and in more general situation in numerical way, it is possible to go beyond the two extreme cases that have been illustrated here *i.e.* the weakly interacting or the totally absorbing situation.

The model presented allows a systematization of different aspects of the hard processes in multiparticle production with a particular attention to the unitarity corrections and suggests also some interpretations in terms of hadron structure. The connection with QCD is not direct as it appears from the fact that the interaction term is quartic while the fundamental QCD interaction term is cubic, in other words the fundamental input is the parton hard scattering, not the branching process.

Acknowledgments

This work has been partially supported by the Italian Ministry of University and of Scientific and Technological Research by means of the *Fondi per la Ricerca scientifica - Università di Trieste*.

References

1. R.J. Glauber, in *Lectures in Theoretical Physics* ed. W.E. Brittin (New York, 1959). M.M.Islam, in *Lectures in Theoretical Physics* vol 9B (New York, 1967).
2. L.L.Amettler, D.Treleani *Int. J. Mod. Phys.* **A3**, 521 (1988). G. Calucci, D. Treleani *Phys. Rev.* **D 41**,3367 (1990). G.Calucci, D.Treleani,*Phys. Rev.* **D 44**,2746 (1991)
3. H.M. Fried *Functional methods and models in QFT, ch.9* M.I.T. Press Cambridge, U.S.A. (1973)
4. J.D. Bjorken, in *Multiparticle dynamics 1994* ed. A.Giovannini, S.Lupia, R.Ugoccioni (Singapore 1995)
5. O. Nachtmann, High-energy collisions and nonperturbative QCD in *Lectures on QCD - Applications* ed. F.Lenz, H.Grießhammer, D.Stoll, (Springer 1997)
6. M. Drees and T. Han, *Phys. Rev. Lett.* **77**, 4142 (1996). F. Abe et al., (CDF Collaboration),*Phys. Rev.* **D 56**, 3811 (1997)
7. G.Calucci, D.Treleani *Nucl.Phys. B(Proc. Suppl.)* **71**,392 (1999)